Riemann International RISM School of Mathematics

NONLINEAR PDES: THEORY AND MODELLING OF REAL PHENOMENA

from 26 to 28 June 2024

Villa Toeplitz, Varese

ON THE OCCASION OF FILIPPO GAZZOLA'S 60^{th} BIRTHDAY

BOOKLET

EVENT SCHEDULE AND PRESENTATIONS ABSTRACTS

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Nonlinear PDEs: theory and modelling of real phenomena

On the occasion of Filippo Gazzola's 60th birthday

RISM, Varese (Italy) – June 26^{th}-28^{th}, 2024

Schedule

	Wednesday 26^{th}	**Thursday 27^{th}**	**Friday 28^{th}**
9:30-10:00	Del Pino	Cianchi	Malchiodi
10:05-10:35	Galdi	Cingolani	Arioli
10:40-11:10	Grillo	Ciraolo	Lamberti
11:15-11:45	*Coffee break*	*Coffee break*	*Coffee break*
11:45-12:15	Ferone	Salani	Trombetti
12:20-12:50	Pata	Weth	Cannarsa
13:00-14:30	*Lunch*	*Lunch*	*Lunch*
14:30-15:00	Musso	Farina	
15:05-15:35	Bonheure	Crasta	
15:40-16:10	Secchi	Moreira dos Santos	
16:15-16:45	*Coffee break*	*Coffee break*	
16:45-17:15	Grunau	Bucur	
17:20-17:50	Sweers	Fragalà	
17:55-18:25	Conti	D'Ambrosio	
		Social dinner	

Abstracts

Periodic and quasiperiodic waves on the sphere

Gianni ARIOLI (Politecnico di Milano)

We construct periodic and quasiperiodic solutions for the nonlinear plate equation on the unit sphere in $\mathbb{R}^3$. Periodic solutions for the nonlinear wave equation are constructed as well. The methods do not involve small parameters. In the case of a cubic nonlinear term, our approach is computer-assisted and thus yields detailed information about each solution. For nonlinearities that are in some sense close to linear, weak solutions are obtained by a variational method.

Behaviour of the hydrodynamic forces acting on a solid close to a wall

Denis BONHEURE (Université Libre de Bruxelles)

I will consider a simple family of fluid-structure interaction problems where a particle interacts with a fluid in a container with boundary. I will explain how to sharply compute the asymptotics of the fluid velocity when the particle is close to the boundary. As a byproduct I will show how to produce sharp estimates on the hydrodynamic forces and the Stokes resistant matrix. The construction is fully variational.

Geometric excess of the fundamental gap

Dorin BUCUR (Université de Savoie)

On convex sets of $\mathbb{R}^N$ the Payne-Weinberger and Andrews-Clutterbuck inequalities provide sharp lower bounds of the fundamental gap of the Laplace operator with Neumann and Dirichlet boundary conditions, respectively, in terms of the diameter. For $N \geqslant 2$ the bounds are not attained and are improved with a geometric term related to the flatness of the convex set. This is a joint work with V. Amato and I. Fragalà.

Long time behaviour of generalised gradient flows via occupational measures

Piermarco CANNARSA (Università di Roma Tor Vergata)

This talk will discuss new methods to study the long time behaviour of the generalised gradient flow (GGF) of a solution, u, of the critical equation for mechanical systems on the d-dimensional flat torus. For this, it is necessary to look at the critical set of u, which turns out to be an attractor for the flow. Moreover, a refined analysis will allow to establish whether GGF approaches regular or singular critical points as t goes to infinity. One crucial tool for this approach is provided by limiting occupational measures, a family of measures that are GGF-invariant (see [Z. Artstein. Invariant measures of differential inclusions applied to singular perturbations. J. Differential Equations, 152(2): 289-307, 1999]). This is a joint work with W. Cheng and C. Mendico.

Second-order regularity for nonlinear elliptic problems

Andrea CIANCHI (Università di Firenze)

Second-order regularity results are established for solutions to second-order elliptic equations and systems, in divergence form, with principal part having Uhlenbeck structure and square-integrable right-hand sides. Such a regularity amounts to the membership in a Sobolev space of the so-called stress-field associated with the elliptic operator. In the case of equations, differential operators depending on anisotropic norms of the gradient are also included. Both local and global estimates are obtained. Global estimates concern solutions to homogeneous Dirichlet problems under minimal regularity assumptions on the boundary of the domain. In particular, no regularity of its boundary is needed if the domain is convex. A critical step in the approach is a sharp pointwise inequality for the involved elliptic operator. This talk is based on diverse joint investigations with C.A. Antonini, A. Kh. Balci, G. Ciraolo, L. Diening, A. Farina and V. Maz'ya.

Weighted Sobolev spaces and Morse estimates for quasilinear elliptic equations

Silvia CINGOLANI (Università di Bari)

There are still several open problems concerning with local Morse theory for functionals in Banach spaces. I will discuss some of them, starting from functionals associated to quasilinear elliptic equations. I will present some advances, including critical group estimates for p-Laplace equations, $p > 2$. The proof relies on new uniform Sobolev inequalities for approximating problems. This is a joint paper with Marco Degiovanni and Berardino Sciunzi.

Quantitative stability results via the method of moving planes

Giulio CIRAOLO (Università di Milano)

The method of the moving planes is a classical tool to prove symmetry properties for overdetermined PDE's boundary value problems and for rigidity problems in geometric analysis. In this talk we give an overview of some recent results related to quantitative studies of the method of moving planes, where quantitative approximate symmetry results are obtained.

A perturbation of the Cahn-Hilliard equation with logarithmic nonlinearity

Monica CONTI (Politecnico di Milano)

In this talk we present some insights about a new perturbation of the Cahn–Hilliard equation, based on an unconstrained theory. This is a joint work with Alain Miranville and Stefania Gatti.

Variational worn stones

Graziano CRASTA (Sapienza Università di Roma)

We introduce an evolution model à la Firey for a convex stone which tumbles on a beach and undertakes an erosion process depending on some variational energy, such as torsional rigidity, principal Dirichlet Laplacian eigenvalue, or Newtonian capacity. Relying on the assumption of existence of a solution to the corresponding parabolic flow, we prove that the stone tends to become asymptotically spherical. Indeed, we identify an ultimate shape of these flows with a smooth convex body whose ground state satisfies an additional boundary condition, and we prove symmetry results for the corresponding overdetermined elliptic problems. Moreover, we extend the analysis to arbitrary convex bodies: we introduce new notions of cone variational measures and we prove that, if such a measure is absolutely continuous with constant density, the underlying body is a ball. This is a joint work with Ilaria Fragalà (Politecnico di Milano).

Liouville theorems for semilinear biharmonic equations and inequalities

Lorenzo D'AMBROSIO (Università di Udine)

We study distributional solutions of semilinear biharmonic equation of the type

$$\Delta^2 u + f(u) = 0 \quad \text{on} \quad \mathbb{R}^N.$$

The analysis is made for general solutions without any assumption on their sign nor on their behaviour at infinity. A relevant role is played by some extensions of the Hardy-Rellich inequalities for general functions (not necessarily compactly supported).

Overhanging solitary waves in the water wave problem

Manuel DEL PINO (University of Bath)

In the classical water wave problem, we construct new overhanging solitary waves by a procedure resembling desingularization of the gluing of constant mean curvature surfaces by tiny catenoid necks. The solutions here predicted have long been numerically detected. This is joint work with Juan Davila, Monica Musso and Miles Wheeler.

Classification results, rigidity theorems and semilinear PDEs on Riemannian manifolds: a P-function approach

Alberto FARINA (Université de Picardie Jules Verne)

We consider solutions to some semilinear elliptic equations on complete noncompact Riemannian manifolds and study their classification as well as the effect of their presence on the underlying manifold. When the Ricci curvature is non-negative, we prove both the classification of positive solutions to the critical equation and the rigidity for the ambient manifold. The same results are obtained when we consider solutions to the Liouville equation on Riemannian surfaces. The results are obtained via a suitable P-function whose constancy implies the classification of both the solutions and the underlying manifold. The analysis carried out on the P-function also makes it possible to classify non-negative solutions for subcritical equations on manifolds enjoying a Sobolev inequality and satisfying an integrability condition on the negative part of the Ricci curvature. This is a joint work with Giulio Ciraolo and Camilla Chiara Polvara.

Symmetrization results for general nonlocal linear elliptic and parabolic problems

Vincenzo FERONE (Università di Napoli Federico II)

We discuss a Talenti-type symmetrization result in the form of mass concentration (*i.e.* integral comparison) for very general linear nonlocal elliptic problems, equipped with homogeneous Dirichlet boundary conditions. In this framework, the relevant concentration comparison for the classical fractional Laplacian can be reviewed as a special case of our main result, thus generalizing previous results obtained in collaboration with B. Volzone. Also a Cauchy-Dirichlet nonlocal linear parabolic problem is considered. The results are contained in a joint paper with G. Piscitelli and B. Volzone.

Riesz inequality for polygons: symmetry and symmetry breaking

Ilaria FRAGALÀ (Politecnico di Milano)

I will discuss counterparts of the classical Hardy-Littlewood and Riesz inequalities when the class of admissible domains is the family of polygons with a fixed number of sides. The latter corresponds to study the polygonal isoperimetric inequality in nonlocal version. Based on a joint work with Beniamin Bogosel and Dorin Bucur.

On self-propulsion by oscillations in a viscous liquid

Giovanni Paolo GALDI (University of Pittsburgh)

A body $\mathcal{B}$ moves in an otherwise quiescent Navier-Stokes liquid, $\mathcal{L}$, filling the entire space outside $\mathcal{B}$. We consider the case when $\mathcal{B}$ is prevented from performing rigid rotations around its center of mass G by the application of a suitable torque, so that its motion is translatory. Denote by $\Omega = \Omega(t)$, $t \in \mathbb{R}$, a one-parameter family of bounded, sufficiently smooth domains of $\mathbb{R}^3$, each one representing the configuration of $\mathcal{B}$ at time t with respect to a frame with the origin at G and axes parallel to those of an inertial frame. We assume that there are no external forces acting on the coupled system $\mathcal{S} := \mathcal{B} \cup \mathcal{L}$ and that the only driving mechanism is a *prescribed* change in shape of Ω with time. The self-propulsion problem that we would like to address can be thus qualitatively formulated as follows. Suppose that $\mathcal{B}$ changes its shape in a given time-periodic fashion, so that, for some $T > 0$ and all $t \in \mathbb{R}$, $\Omega(t + T) = \Omega(t)$. Then, find necessary and sufficient conditions on the map $t \mapsto \Omega(t)$ securing that $\mathcal{B}$ self-propels, namely, the center of mass G covers any given finite distance in a finite time.

Stochastic completeness and nonlinear elliptic and parabolic equations

Gabriele GRILLO (Politecnico di Milano)

Stochastic completeness of a manifold amounts to asking that the heat semigroup preserves probability, and is hence a property linked to a linear evolution equation. The property may or may not hold, and several geometric and analytic conditions are available in order to discern whether it is satisfied, or not. We show here that stochastic incompleteness can be characterized in terms of existence of (at least one, hence infinitely many) solutions to a large class of nonlinear elliptic equations, and to non-uniqueness of solutions to the filtration equation, with very general nonlinearities. This is a joint work with K. Ishige, M. Muratori and F. Punzo.

A biharmonic analogue of the Alt-Caffarelli problem

Hans-Christoph GRUNAU (Otto-von-Guericke-Universität Magdeburg)

We study a natural biharmonic analogue of the classical Alt-Caffarelli problem under so called Navier boundary conditions. We show existence, basic properties and $C^{1,\alpha}$-regularity of minimisers. In the case that the domain is a ball and the boundary data are radial we also obtain a symmetry result. We find this remarkable because the problem under investigation is of higher order. Computing radial minimisers explicitly we find that the obtained regularity is optimal. The talk is based upon joint work with Marius Müller (Augsburg).

Steklov vs. Steklov: a fourth-order affair related to the Babuska paradox

Pier Domenico LAMBERTI (Università di Padova)

We discuss two Steklov problems for the biharmonic operator: the classical one (DBS – Dirichlet Biharmonic Steklov) introduced by Kuttler and Sigillito in 1968, and a modified one (MDBS – Modified Biharmonic Steklov) proposed in a joint paper with Alberto Ferrero in 2019. We examine their stability with respect to domain perturbations in connection with paradoxical behaviors of the Babuska type, possibly involving a "strange curvature". Additionally, we highlight recent results that indicate the modified Steklov problem is better behaved with respect to certain shape optimization problems. Our analysis suggests that the names of the two problems could be better swapped.

Sharp bounds on the Nusselt number in Rayleigh-Bénard convection

Andrea MALCHIODI (Scuola Normale Superiore di Pisa)

We analyze the problem in fluid dynamics of deriving bounds for heat transportation in the infinite Prandtl number limit. Due to a maximum principle property for the temperature, this amounts to proving a-priori bounds for horizontally-periodic solutions of a fourth-order equation in a strip of large width with both Dirichlet and Neumann data. We obtain such bounds using Fourier analysis, integral representations, and a bilinear estimate due to Coifman and Meyer which uses the Carleson measure characterization of BMO functions by Fefferman. This is a joint work with S. Chanillo.

Spectral partition problems with volume and inclusion constraints

Ederson MOREIRA dos SANTOS (Universidade de São Paulo)

In this talk we discuss a class of spectral partition problems with a measure constraint, for partitions of a given bounded connected open set. We establish the existence of an optimal open partition, showing that the corresponding eigenfunctions are locally Lipschitz continuous, and obtain some qualitative properties for the partition. The proof uses an equivalent weak formulation that involves a minimization problem of a penalized functional where the variables are functions rather than domains, suitable deformations, blowup techniques and a monotonicity formula. This is a joint work with Pêdra Andrade, Makson Santos and Hugo Tavares (IST-Lisboa).

Leapfrogging vortex rings for Euler equations

Monica MUSSO (University of Bath)

We consider the Euler equations for incompressible fluids in 3-dimension. A classical question that goes back to Helmholtz is to describe the evolution of vorticities with a high concentration around a curve. The work of Da Rios in 1906 states that such a curve must evolve by the so-called "binormal curvature flow". Existence of true solutions whose vorticity is concentrated near a given curve that evolves by this law is a long-standing open question that has only been answered for the special case of a circle travelling with constant speed along its axis, the thin vortex-rings, and of a helical filament, associated to a translating-rotating helix. In this talk I will consider the case of two vortex rings interacting between each other, the so-called leapfrogging. The results are in collaboration with J. Davila, M. del Pino and J. Wei.

Abstract damped wave equations: the optimal decay rate

Vittorino PATA (Politecnico di Milano)

We discuss the exponential decay rate of the semigroup generated by the abstract damped wave equation. We prove that, for general dampings, the decay is dictated by the spectrum of the infinitesimal generator of the semigroup. In particular, we show that the semigroup fulfills the strong spectrum determined growth property, except in the resonant case, where the exponential rate is polynomially penalized. In all cases, the decay rate is the best possible allowed by the theory.

Concavity principles for the principal frequency in Gauss space

Paolo SALANI (Università di Firenze)

I will prove a Brunn-Minkowski inequality for the Dirichlet eigenvalue of the Ornstein-Uhlenbeck operator and the log-concavity of the associated eigenfunction of a convex domain. The talk is based on a joint work with A. Colesanti, E. Francini and G. Livshyts.

The two-dimensional plasma-vacuum interface problem in ideal MHD

Paolo SECCHI (Università di Brescia)

In this talk we consider the two-dimensional plasma-vacuum interface problem in ideal compressible magnetohydrodynamics (MHD). This is a hyperbolic-elliptic coupled system with a characteristic free boundary. In the plasma region the 2D planar flow is governed by the hyperbolic equations of ideal compressible MHD, while in the vacuum region the magnetic field obeys the elliptic system of pre-Maxwell dynamics. At the free interface moving with the velocity of plasma particles, the total pressure is continuous and the magnetic field on both sides is tangent to the boundary. The plasma-vacuum system is not isolated from the outside world, since it is driven by a given surface current which forces oscillations onto the system. We present our result about the local-in-time existence and uniqueness of solutions to the nonlinear free boundary problem, provided that the plasma magnetic field or the vacuum magnetic field is non-zero at each point of the initial interface. The proof follows from the analysis of the linearized MHD equations in the plasma region and the elliptic system for the vacuum magnetic field, suitable tame estimates in Sobolev spaces for the full linearized problem, and a Nash-Moser iteration. This is a joint work with A. Morando (Brescia), Y. Trakhinin (Novosibirsk), P. Trebeschi (Brescia) and D. Yuan (Beijing Normal University).

On a formula for all sets of constant width in 3D

Guido SWEERS (Universität zu Köln)

$\Omega \subset \mathbb{R}^n$ is a set of constant width R_Ω, when its directional width

$$d(\omega) = \sup_{x\in\Omega} \langle x, \omega\rangle - \inf_{x\in\Omega} \langle x, \omega\rangle$$

is constant: $d(\omega) = R_\Omega$ for all $\omega \in \mathbb{S}^{n-1}$. For dimension 2 the theorem of Blaschke-Lebesgue (1914) states that any set Ω of constant width satisfies

$$V_2(\Omega) \geqslant \frac{1}{2}\left(\pi - \sqrt{3}\right) R_\Omega^2$$

and equality only holds, when Ω is a Reuleaux triangle. $V_2(\Omega)$ is the area ($2d$-volume) of Ω. A similar result in dimension 3 is still open. Bonnesen and Fenchel conjectured in 1934 that the optimal domain is the Meissner body, which would give

$$V_3(\Omega) \geqslant \left(\frac{2}{3} - \frac{1}{4}\sqrt{3}\arccos\left(\frac{1}{3}\right)\right) R_\Omega^3.$$

The conjecture still seems open. Part of the problem was that not even an explicit constructive formula for all bodies of constant width was known. With Bernd Kawohl we found at least such a formula. Although this formula allows a variational formulation of $R_\Omega^{-3} V_3(\Omega)$, minimization leads to a differential equation with second and fourth order derivatives. Moreover, it has long been proven that a minimizing $\partial\Omega$ is nonsmooth in either x_ω or $x_{-\omega}$ for each $\omega \in \mathbb{S}^2$, where

$$\langle x_\omega, \omega\rangle = \sup_{x\in\Omega} \langle x, \omega\rangle \text{ and } \langle x_{-\omega}, \omega\rangle = \inf_{x\in\Omega} \langle x, \omega\rangle .$$

I hope to explain the formula for 3d-bodies of constant width and if unsuccessful, then one might still have a look at the corresponding paper:

- B. Kawohl, G. Sweers, On a formula for all sets of constant width in 3D, J. Geom. Anal. 34, 197, 38pp (2024).

A Talenti comparison result for the Neumann boundary value problem

Cristina TROMBETTI (Università di Napoli Federico II)

In light of a recent paper that extended Talenti's a priori estimate for solutions to elliptic problems from Dirichlet to Robin boundary conditions, we show that the Neumann case can also be considered. This leads to the derivation of optimal bounds that appear to be previously unknown.

Rotating waves in nonlinear media and critical degenerate Sobolev inequalities

Tobias WETH (Goethe-Universität Frankfurt)

I will report on recent work on rotating wave solutions to the Dirichlet problem for a class of nonlinear wave equations in radial domains. Depending on the prescribed angular velocity of the rotation, the ansatz for rotating waves leads to a Dirichlet problem for a semilinear elliptic or degenerate elliptic equation. We show that this problem is governed by an associated critical degenerate Sobolev inequality in the half space. After proving this inequality and the existence of associated extremal functions, we then deduce necessary and sufficient conditions for the existence of ground state solutions. Moreover, we analyze under which conditions on the underlying parameters these ground states are nonradial and therefore give rise to truly rotating waves. This is joint work with Joel Kübler (Frankfurt).

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Thanks for your participation

Speakers

Gianni Arioli (Politecnico di Milano)
Denis Bonheure (Université Libre de Bruxelles)
Dorin Bucur (Université de Savoie)
Piermarco Cannarsa (Università di Roma Tor Vergata)
Andrea Cianchi (Università di Firenze)
Silvia Cingolani (Università di Bari)
Giulio Ciraolo (Università di Milano)
Monica Conti (Politecnico di Milano)
Graziano Crasta (Sapienza Università di Roma)
Lorenzo D'Ambrosio (Università di Udine)
Manuel Del Pino (University of Bath)
Alberto Farina (Université de Picardie Jules Verne)
Vincenzo Ferone (Università di Napoli Federico II)
Ilaria Fragalà (Politecnico di Milano)
Giovanni Paolo Galdi (University of Pittsburgh)
Gabriele Grillo (Politecnico di Milano)
Hans-Christoph Grunau (Otto-von-Guericke-Universität Magdeburg)
Pier Domenico Lamberti (Università di Padova)
Andrea Malchiodi (Scuola Normale Superiore di Pisa)
Enzo Mitidieri (Università di Trieste)
Ederson Moreira dos Santos (Universidade de São Paulo)
Monica Musso (University of Bath)
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