Abstracts

Bifurcation for a sharp interface generation problem

Emilio ACERBI
Università di Parma

Abstract. As opposed to the widely studied bifurcation phenomena for maps or PDE problems, we are concerned with bifurcation for stationary points of a non-local variational functional defined not on functions but on sets of finite perimeter. We consider the sharp interface limit of the FitzHugh-Nagumo energy functional in a flat torus in R^2, for which lamellar stationary points exist and their stability range is well understood, and we prove that when the lamella loses its stability bifurcation occurs, leading to a two-dimensional branch of nonplanar stationary points. The same results are obtained for multi-lamellae, which exhibit a transition from supercritical to subcritical bifurcation as the torus size increases. To the best of our knowledge, bifurcation for nonlocal problems in a geometric measure theoretic setting is an entirely new result.

Optimal coefficients for elliptic PDEs

Giuseppe BUTTAZZO
Università di Pisa

Abstract. The goal is to present an optimization problem related to elliptic PDEs of the form $-\mathrm{div}(a(x)\nabla u) = f$ with Dirichlet boundary condition on a given domain Ω. The coefficient $a(x)$ has to be determined, in a suitable given class of admissible choices, in order to optimize a given criterion. The first deal with the case when the cost is the so-called elastic compliance, and then we discuss the more general case when the problem is written as an optimal control problem.

A fully second-order scheme for nonlinear peridynamics on manifolds

Alessandro COCLITE
Politecnico di Bari

Abstract. Peridynamics elasticity theories have become widespread in various research areas due to their ability in modeling discontinuities formation and evolution. Due to its physical intuitiveness, bond-based peridynamics is widely employed in numerical simulations notwithstanding some modeling limitations. In this work, the problem of approximating the solution of the nonlinear peridynamics wave equation on arbitrarily–shaped two- dimensional (2D) smooth manifolds is proposed. A finite difference fully second-order scheme on a triangular mesh is proposed in [1]. Here, due to the manifold arbitrarily shaping, computing the distance between two non-adjacent points arises as a tricky problem. A routing procedure is implemented by reinterpreting the computational mesh as a non-oriented graph thus returning a suitable and general method. The time integration of the peridynamics equation is demanded to a $P-(EC)^k$ formulation of the implicit β-Newmark scheme. Local Dirichlet conditions are applied on external boundaries. A suitable choice for the micromodulus function is adopted to recover the local elasticity limit for vanishing peridynamics horizon and, at the same time, to eliminate the softening near boundaries. Abilities and limitations of the proposed procedure are critically analyzed by simulating several benchmark tests. This talk is based on joint works with G. M. Coclite, N. Dimola, G. Fanizza, F. Maddalena, and T. Politi [1, 2, 3, 4].

References

[1] Coclite, A., Coclite, G. M., Maddalena, F., and Politi, T. (2022). A numerical frame- work for nonlinear peridynamics on two-dimensional manifolds based on implicit $P-(EC)^k$ schemes. SIAM Journal on Numerical Analysis. 62.2 (2024): 622-645.

[2] Dimola, N., Coclite, A., Fanizza, G., and Politi, T. (2022). Bond-based peridynamics, a survey prospecting nonlocal theories of fluid-dynamics. Advances in Continuous and Discrete Models, 2022 (1), 1-26.

[3] Coclite, A., Coclite, G. M., Fanizza, G., and Maddalena, F. (2023). Dispersive effects in two-and three-dimensional peridynamics. Acta Applicandae Mathematicae 187.1 (2023), p. 13.

[4] Coclite, A. and Dimola, N. (2024). On the micromodulus function in peridynamics for preserving local-elasticity limit. In preparation.

Homogenisation of free discontinuity problems

Gianni DAL MASO
SISSA, Trieste

Abstract. We study deterministic and stochastic homogenisation problems for free discontinuity functionals under new hypotheses on the surface energies. The results are based on a compactness theorem with respect to Gamma-convergence, on the characterisation of the integrands of the Gamma-limit by means of limits of minimum values of some auxiliary minimum problems on small cubes, and on the subadditive ergodic theorem for the stochastic part.

Optimal transport of measures via autonomous vector fields

Nicola DE NITTI
EPFL Lausanne

Abstract. We study the problem of transporting one probability measure to another via an autonomous vector field. We rely on tools from the theory of optimal transport. In one space-dimension, we construct a suitable autonomous vector field that realizes the (unique) monotone transport map as the time-1 map of its flow. We then use Sudakov's disintegration approach to deal with the multi-dimensional case by reducing it to a family of one-dimensional problems. This talk is based on a joint work with X. Fernández-Real.

Boundary continuity of nonlocal minimal surfaces in domains with singularities

Serena DIPIERRO
University of Western Australia

Abstract. The stickiness phenomenon is one of the most characteristic features of nonlocal minimal surfaces. In this talk, we will present how this property is influenced by corners and singularities of the domain under consideration.

The geometric size of the fundamental gap

Ilaria FRAGALÀ
Politecnico di Milano

Abstract. The fundamental gap conjecture proved by Andrews and Clutterbuck in 2011 provides the sharp lower bound for the di!erence between the first two Dirichlet Laplacian eigenvalues in terms of the diameter of a convex set in the N-dimensional Euclidean space. The question concerning the rigidity of the inequality, raised by Yau in 1990, was left open. In this talk I will discuss some very recent results which answer affirmatively such question and, going beyond, strengthen Andrews-Clutterbuck inequality, by quantifying geometrically the excess of the gap compared to the diameter in terms of flatness. The results are contained in a paper in collaboration with Vincenzo Amato and Dorin Bucur.

Phase-field approximation of cohesive fracture energies

Matteo FOCARDI
Università di Firenze

Abstract. The functional analytic setting of various variational models in Fracture Mechanics requires the use of classes of functions with a set of discontinuities of codimension one. The di!culty of finding good discretization for such classes of functions makes the direct numerical simulation of those variational problems challenging and highly problematic. For this reason, numerous regularizations have been proposed, the most successful of which are phase-field functionals. These elliptic regularizations were first introduced and analyzed in the work of Ambrosio and Tortorelli for approximating the Mumford-Shah energy in image segmentation, and were inspired by a now classical example in phase transition by Modica and Mortola. Ambrosio and Tortorelli type approximations have become very popular both in the communities of Calculus of Variations and of Computational Mechanics to address a number of problems in applied sciences, especially in brittle fracture. In the talk, we will comment on some of those phase-field models, starting with Ambrosio and Tortorelli's, and which eventually led to variants useful for approximating cohesive energies in Fracture Mechanics.

Variational linearization of obstacle problems in nonlinear elasticity

Francesco MADDALENA
Politecnico di Bari

Abstract. An energy functional for the obstacle problem in linear elasticity is obtained as a variational limit of nonlinear elastic energy functionals. The problem concerns the equilibrium of an elastic body subject to pure traction load under a unilateral constraint representing the rigid obstacle. For a class of loads unfitting the geometry of the system body-obstacle the corresponding variational limit turns out to be different from the classical Signorini problem in linear elasticity. However, if the force field acting on the body fulfils an appropriate geometric admissibility condition, we can show coincidence of minima. The analysis developed here provides a rigorous variational justification of the Signorini problem in linear elasticity, together with an accurate analysis of the unilateral constraint. This is a joint work with Danilo Percivale and Franco Tomarelli.

References

[1] F. Maddalena, D. Percivale, F. Tomarelli, The gap between linear elasticity and the variational limit of finite elasticity in pure traction problems, Arch. Rational Mech.Anal., 234 (2019), 10911120. https://doi.org/10.1007/s00205-019-01408-2.

[2] F. Maddalena, D. Percivale, F. Tomarelli, A new variational approach to linearization of traction problems in elasticity, J. Optim. Theory Appl., 182 (2019), 383403, https://doi.org/10.1007/s10957-019-01533-8

[3] F. Maddalena, D. Percivale, F. Tomarelli, Signorini problem as a variational limit of obstacle problems in nonlinear elasticity , Mathematics in Engineering, 6(2): 261304, http://dx.doi.org/10.3934/mine.2024012

Almansi Decomposition and Expansion of a Polyharmonic Function Near a Crack-Tip

Antonio LEACI
Università del Salento

Abstract. We have studied polyharmonic functions in 2-dimensional open sets with a flat crack: for these functions we show a decomposition of Almansi-type and make explicit the coefficients of a strongly converging expansion near the crack-tip. Results of this kind are relevant in the study of Blake and Zisserman functional. Moreover the analysis either of crack-tip or crease-tip is met in the study of variational models for image segmentation, inpainting and denoising. Such kind of results may have a wider range of application than image analysis, namely the analysis of singularities for free discontinuity problems and crack-tips in planar elasticity, polyharmonic functions in open sets. This is a joint work with Michele Carriero and Franco Tomarelli.

Global Hölder continuity of solutions to quasilinear equations with Morrey data

Dian PALAGACHEV
Politecnico di Bari

Abstract. We will discuss regularity issues regarding general quasilinear divergence-form coercive equations

$$\operatorname{div}\big(\mathbf{a}(x,u,Du)\big) = b(x,u,Du),$$

whose prototype is the m-Laplacian equation. The nonlinear terms are given by Carathéodory functions and satisfy controlled growth structure conditions in u and Du, while their behaviour with respect to x is modeled in Morrey spaces. The fairly non-smooth boundary of the underlying domain is supposed to support a capacity density condition that allows domains with exterior corkscrew property.

Global boundedness and Hölder continuity up to the boundary will be shown for the weak solutions of such equations, generalizing this way the classical L^p-results of Ladyzhenskaya and Ural'tseva ([1, 2, 3]) to the settings of the Morrey spaces.

Some applications to nonlinear Calderón–Zygmund theory in Morrey spaces, and generalizations to componentwise coercive systems, will be discussed as well.

The results presented ([4]) are obtained in collaboration with S.-S. Byun (Seoul) and P. Shin (Suwon).

References

[1] O.A. Ladyzhenskaya and N.N. Ural'tseva, On the Hölder continuity of the solutions and the derivatives of linear and quasi-linear equations of elliptic and parabolic types, *Trudy Mat. Inst. Steklov* **73** (1964), 172–220.

[2] O.A. Ladyzhenskaya and N.N. Ural'tseva, *Linear and Quasilinear Equations of Elliptic Type,* 2nd Edition revised, Nauka, Moscow, 1973.

[3] O.A. Ladyzhenskaya and N.N. Ural'tseva, A survey of results on the solvability of boundary value problems for uniformly elliptic and parabolic second-order quasilinear equations having unbounded singularities, *Russian Math. Surveys* **41** (1986), no. 5, 1–31.

[4] S.-S. Byun, D.K. Palagachev and P. Shin, Global Hölder continuity of solutions to quasilinear equations with Morrey data, *Commun. Contemp. Math.* **24**(2023), Art. no. 2150062.

Microscopical justification of solid-state wetting and dewetting

Paolo PIOVANO
Politecnico di Milano

Abstract. The Winterbottom problem, that consists in determining the equilibrium shape of crystalline drops resting on a flat substrate, is considered, and the related continuum model is derived from atomistic models by means of a discrete-to-continuum passage performed by Gamma-convergence. Previous results for the Wulff shape are generalized and, as a byproduct of the analysis, effective expressions for the drop anisotropy and the substrate wettability are characterized in terms of atomistic Heitmann-Radin potentials. Moreover, a threshold condition is determined distinguishing the wetting regime, in which the discrete minimizers are explicitly characterized as one-atom thick partial layers, from the dewetting regime, in which, by proving a conservation of mass, proper scalings of the minimizers are shown to converge to a bounded minimizer of the Winterbottom model. Those results we obtained in collaboration with Igor Velčić (University of Zagreb, Croatia).

The superposition principle for measure-valued solutions to the continuity equation

Giuseppe SAVARÈ
Università Bocconi Milano

Abstract. We present an overview of results concerning the representation of time- dependent measure-valued solutions to the continuity equation by a superposition (characterized by a probability measure) of absolutely continuous or BV curves. The case when the velocity field is not absolutely continuous with respect to the evolving measures is more subtle and requires a localized reparametrization technique, which extends to the mean field setting a typical approach to rate-independent evolutions. (In collaboration with Stefano Almi and Riccarda Rossi).

Control and Machine Learning

Enrique ZUAZUA
FAU, Erlangen–Nürnberg

Abstract. Control theory and Machine Learning share common objectives, as evident in Norbert Wiener's definition of "Cybernetics" as "The science of control and communication in animals and machines." The synergy between these fields is reciprocal. Control theory tools enhance our comprehension of the efficacy of certain Machine Learning algorithms and offer insights for their enhancement. However, this often bounces intricate queries back. The interplay between Control and Machine Learning opens up a new captivating scientific lanscape to be explored but this can be a labyrinthine task. And this is part of the overall ambitious program of developing Digital Twins technologies. In this talk, we will present some of the contributions from our team at the interface between Control and Machine Learning that can contribute to this ambitious and complex task. We will in particular discuss some neural network architectures, whose success for Supervised Learning can be understood from a control perspective and explain how their dimension and complexity can be minimized. We will also present some challenging open problems.

Mixed dimensional PDEs: analysis, approximation and applications

Paolo ZUNINO
Politecnico di Milano

Abstract. Mixed-dimensional PDEs represent coupled problems involving lower dimensional manifolds or sub- manifolds of one or many domains. Arising from applications of paramount importance in the sciences, including but not limited to materials science, geo- and the life sciences, these problems have progressively attracted the attention of various communities of mathematicians.

We focus on mixed dimensional PDEs on three-dimensional (3D) and one-dimensional domains, giving rise to a 3D-1D coupled problem. Such a problem is not well investigated yet from the standpoint of mathematical analysis and numerical approximation, although it arises in applications of paramount importance such as microcirculation, flow through perforated media and the study of reinforced materials, just to make a few examples.

We address this mathematical problem within a general framework, designed to formulate and approximate coupled PDEs on manifolds with heterogeneous dimensionality using topological model reduction tools. The main diffculty consists of the ill-posedness of restriction operators (such as the trace operator) applied on manifolds with co-dimension larger than one. We will overcome this challenge by means of nonlocal restriction operators that combine standard traces with mean values of the solution on low dimensional manifolds. This new approach has the fundamental advantage of enabling the approximation of the problem using Galerkin projections on Hilbert spaces, which can not be otherwise applied because of regularity issues.

Then, we discuss the numerical approximation of the problem in the framework of the finite element method. The main challenge is the discretization of the interface conditions between 3D and 1D problems, addressed using the Lagrange multiplier method. Several options are proposed, analyzed, and compared, with the purpose of determining a good balance between the mathematical properties of the discrete problem and flexibility of implementation of the numerical scheme. Furthermore, combining the numerical error analysis with the model reduction approach, the concurrent modeling and discretization errors in the approximation of the original fully dimensional problem can be estimated.

The results are supported by evidence based on numerical experiments. The application of this technique to fluid-structure interaction and the modeling of reinforced materials will be discussed.