

NEW TRENDS AROUND PROFINITE GROUPS 2022

Sep 14 – Sep 16

Abstract Booklet

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Sep 14— Afternoon Presentations

Pavel Zalesskii, Universidade de Brasilia

14:30 - 15:30

Combinatorial theory of pro- p groups.

Free product with amalgamation and HNN-extension are two main constructions of combinatorial group theory. I shall discuss these two constructions in the category of pro- p groups, presenting results on splittings of pro- p groups as an amalgamated free pro- p product or a pro- p HNN-extension and relating them with pro- p version of Bass-Serre's theory of groups acting on trees. I shall also compare the pro- p results with similar results for abstract groups.

Keywords: pro- p group, Bass-Serre theory, amalgam, HNN-extension.

Christian Maire, Université de Franche-Comté

16:00 - 17:00

Construction of maximal unramified p -extensions of given Galois groups.

About ten years ago Ozaki showed the following result: Given a finite p -group G , there exists a totally imaginary number field K for which the Galois group of its Hilbert p -class field tower is isomorphic to G . In a recent work with Farshid Hajir (UMASS) and Ravi Ramakrishna (Cornell University), we have revisited and simplified Ozaki's proof, which allowed us to relax the condition on the signature of K and to control the degree and the ramification of K/Q . In this talk, I will give the key elements of our proof, and try to bring out the notion of Minkowski units. If time permits, I will address the question of the Minkowski units when the p -tower is infinite.

Keywords: Galois group, unramified p -extension, Hilbert p -class field tower.

Mark Shustermann, Harvard University

17:30 - 18:30

Counting relations in profinite groups.

We discuss recent results and emerging questions obtained by applying Lubotzky's formula for the number of relations to several finitely generated profinite groups that are of interest in arithmetic, geometry, and topology.

Keywords: finitely generated, profinite groups, relations.

Sep 15 — Morning Presentations

Tamar Bar-On, University of Oxford

09:00 - 10:00

Profinite completion of free profinite groups.

The profinite completion of a free profinite group on an infinite set of generators is a profinite group of greater rank. However, it is still not known whether it is a free profinite group too. We will discuss this question and show some partial results, such as the case of pro- p groups.

Keywords: profinite completion, free profinite group, pro- p group.

Conchita Martínez-Pérez, Universidad de Zaragoza

10:30 - 11:30

Lie algebras and groups: cohomological properties.

The study of the interactions between Lie algebras and groups is a classical subject in mathematics. In this talk, we will review some recent results about these interactions, focusing in properties of cohomological nature. This is a joint work with Dessislava Kochloukova.

Keywords: Lie algebra, Lie group, cohomology.

Joshua Maglione, Universität Magdeburg
12:00 - 13:00

Igusa zeta functions and flag Hilbert-Poincaré series of hyperplane arrangements.

We define a class of multivariate rational functions associated with hyperplane arrangements called flag Hilbert-Poincaré series. We show how these rational functions are connected to local Igusa zeta functions and class counting zeta functions for certain graphical group schemes studied by Rossmann and Voll. We report on a general self-reciprocity result and explore other connections within algebraic combinatorics. This is joint work with Christopher Voll and with Lukas Kühne.

Keywords: rational function, Hilbert-Poincaré series, Igusa zeta function.

Sep 15 — Afternoon Presentations

Matteo Vannacci, Universidad del País Vasco
14:30 - 15:30

Weil zeta functions of representations over finite fields: abscissae of convergence.

In this talk we will encode the number of representations over finite fields of a profinite group G into a zeta-function and we will investigate how analytic properties of this zeta-function connect to properties of the original group G and its completed group-ring. In particular, we will give estimates for the abscissae of convergence of various "free" objects: e.g. free pro- p , pro-nilpotent and pro-solvable groups. This is joint work with Ged Corob-Cook and Steffen Kionke.

Keywords: profinite group, representation, finite field, zeta function.

Ilij Snopce, Universidade Federal do Rio de Janeiro
16:00 - 17:00

Frattini-resistance and maximal pro- p Galois groups.

Given a pro- p group G , denote by $\Phi(G)$ the Frattini subgroup of G . We call a pro- p group G Frattini-resistant if for all finitely generated subgroups H and K of G , $\Phi(H) \leq \Phi(K)$ implies $H \leq K$. I will talk about Frattini-resistant pro- p groups and their relation with maximal pro- p Galois groups. This talk is based on a joint work with Slobodan Tanushevski.

Keywords: pro- p -group, Frattini resistant, maximal pro- p Galois group.

Karthika Rajeev, Universität Bielefeld
17:30 - 18:30

On the Basilica operation.

The classical Basilica group is a well-studied example of a group acting on the binary rooted tree. Inspired by the classical Basilica group, we introduced a general construction that yields a new family of Basilica groups from any group of automorphisms of a rooted tree. We investigate the properties of the groups preserved under the Basilica operation. Furthermore, we develop new techniques for computing the Hausdorff dimension of groups with strong self-similarity features. Explicit computations will be presented for groups that resemble the classical Basilica group. This is joint work with Jan Moritz Petschick.

Keywords: rooted tree, automorphism, Basilica operation, Hausdorff dimension.

Sep 16 — Morning Presentations

Jon González-Sánchez, Universidad del País Vasco

09:00 - 10:00

Zeta functions associated to irreducible representations of profinite groups over number fields.

Let G be a finitely generated profinite group and K a number field (e.g. the field of rational numbers), then the number a_n of irreducible representations of G over the field K of a given degree n is finite. If G is p -adic analytic the growth of a_n is polynomial and one can associate to the G and K a zeta function. In this talk we will explain how can be used the Kirillov's Orbit Method and Brauer-Clifford Theory to study such zeta functions.

Keywords: profinite group, representation, number field, zeta function.

Paula Lins de Araujo, University of Lincoln

10:30 - 11:30

Zeta functions and points on elliptic curves.

In this talk we discuss zeta functions introduced by Grunewald, Segal and Smith [1988] that encode the number of normal subgroups of nilpotent groups. One of the major questions raised by them is how the local factors at a prime p vary for different p . Many of the examples showed a uniform behaviour as the prime varied. When considering nilpotent groups associated to elliptic curves, du Sautoy [2001, 2002] provided examples of zeta functions whose local factors do not behave uniformly when one varies the primes considered. Inspired by du Sautoy's work, we investigate how zeta functions of nilpotent groups associated to different elliptic curves reflect particular properties of these curves. This is a joint work with Victoria Cantoral-Farfán.

Keywords: zeta function, local factor, nilpotent group, elliptic curve.

Benjamin Klopsch, Universität Düsseldorf

12:00 - 13:00

Finite axiomatizability of the rank and the dimension of p -adic analytic pro- p groups.

Recently, Nies, Segal and Tent initiated an investigation into finite axiomatizability (in the sense of model theory) for profinite groups. Among several other types of groups, they consider in some detail the class of p -adic analytic pro- p groups. In joint work with Martina Conte, we have now answered some questions regarding the rank and the dimension of p -adic analytic groups which arise naturally from the work of Nies, Segal and Tent. For instance, we prove that for every prime p and every positive integer r there is a sentence $\sigma_{p,r}$ in the first-order language of groups such that for every pro- p group G the following are equivalent: (i) G is a model of $\sigma_{p,r}$, (ii) G has rank r . In my talk I will explain briefly the necessary background and then discuss why this theorem and related results hold true. Time permitting I will further present natural generalisations and I will indicate some open problems.

Keywords: pro- p group, p -adic analytic group, finite axiomatizability, first-order language