DinAmicI VII

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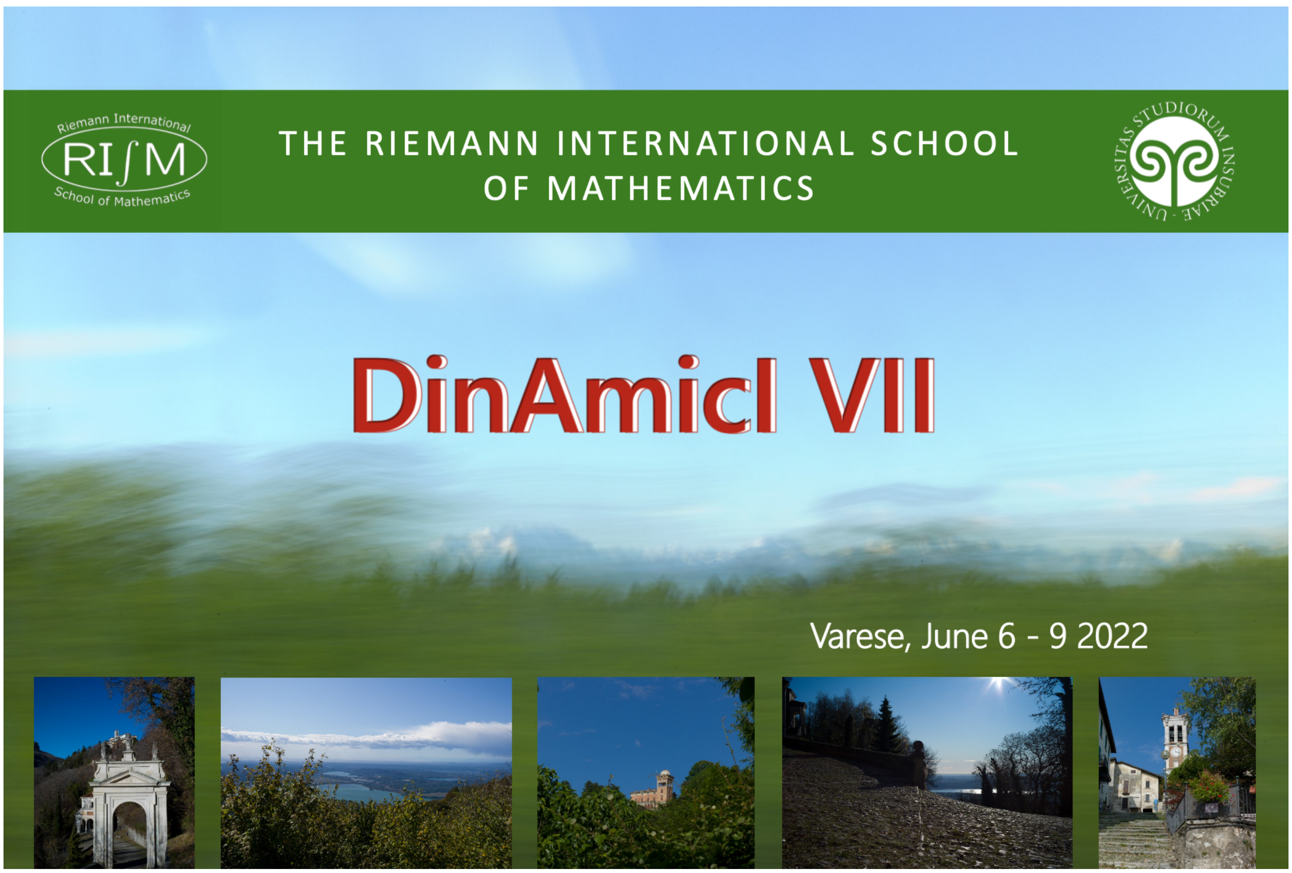

Abstracts of the talks

Jon Aaronson (Tel Aviv University, Tel Aviv, IL)

Some infinite ergodic renewal theory

I'll begin with a tied-down, distributional limit for renewal processes (as defined in Chung's 1966 book) which satisfy the strong renewal theorem (Caravenna-Doney, Elect. J. Prob. 2019) and then prove a "weak Cesaro" version for pointwise dual ergodic transformations with suitably regularly varying return sequences. "Strong Cesaro" versions hold for such transformations admitting Gibbs-Markov return time processes including certain intermittent interval maps with regularly varying Taylor expansions at indifferent fixed points.
Joint work with Toru Sera (`arXiv:1910.09846 & 2104.12006`).

Francesco Cellarosi (Queen's University, Kingston, ON, CA)

The Jacobi theta function near the real line

The Jacobi theta function is a function of two complex variables, τ (with positive imaginary part) and z. As τ approaches the real line, the function may blow up. In a joint work with Tariq Osman (Queen's University), we find a uniform bound for generalised Weyl sums using homogeneous dynamics. As a consequence, we find affine curves $z = z(\tau)$ along which the blow up in the Jacobi theta function can be uniformly controlled.

Emilio Corso (ETH Zurich, Zurich, CH)

Folding dilating shapes: a journey from Euclidean to hyperbolic geometry

What happens to a progressively dilating body when folding the space in which it lives? For a start, we shall examine the problem in a Euclidean context, surveying results of Randol and Strichartz through the language of Rajchman measures and the quantitative viewpoint of Fourier dimension. Afterwards, we take up the same question in hyperbolic spaces, briefly overviewing the equidistribution theorems of Randol and Sarnak while focusing instead on their upgrade to unit tangent bundles, due originally to Margulis and later refined quantitatively in our joint work with Ravotti.

Marian Gidea (Yeshiva University, New York, NY, USA)

Global effect of non-conservative perturbations on homoclinic orbits to a normally hyperbolic invariant manifold

We consider a mechanical system consisting of a rotator and a pendulum coupled via a small, time-periodic Hamiltonian perturbation, and subject to an additional, non-conservative perturbation. The system exhibits a normally hyperbolic invariant manifold (NHIM). The homoclinic orbits to the NHIM can be described via the scattering map, which gives the future asymptotic of an orbit as a function of the past asymptotic. We provide explicit formulas, in terms of convergent integrals, for the perturbed scattering map. As an application, we consider the particular case when the additional perturbation is dissipative. One would expect the resulting system to experience energy loss. Surprisingly, by interspersing the outer dynamics along homoclinic orbits with the inner dynamics restricted to the NHIM, one can achieve a significant growth in energy over time.

Filippo Giuliani (Politecnico di Milano, Milano, IT)

Arnold diffusion in Hamiltonian systems on infinite lattices

In this talk we present a recent result about the existence of transfer of energy orbits in a chain of infinitely many weakly coupled pendulums. The model system is posed on an infinite lattice (of any dimension) with formal or convergent Hamiltonian. We develop geometric and functional tools to perform an Arnold diffusion mechanism in infinite dimensional phase spaces. In this way we construct solutions that move the energy of pendulums along any prescribed path.
This is joint work with M. Guardia.

Marcel Guardia (Universitat Politècnica de Catalunya, Barcelona, ES)

Oscillatory motions and symbolic dynamics in the three body problem

Consider the three body problem with positive masses m_0, m_1 and m_2. In 1922 Chazy classified the possible final motions the three bodies may possess, that is, the behaviors the bodies may have when time tends to infinity. One of them are what is known as oscillatory motions, that is, solutions of the three body problem such that the liminf (as time tends to infinity) of the relative positions between bodies is finite whereas the limsup is infinite. That is, solutions for which the bodies keep oscillating between an increasingly large separation and getting closer together. The first result of existence of oscillatory motions was provided by Sitnikov for a Restricted Three Body Problem, called nowadays Sitnkikov model. His result has been extended to several Celestial Mechanics models but always with rather strong assumptions on the values of the masses. In this talk I will explain how to construct oscillatory motions for the three body problem for any values m_0, m_1 and m_2 (except for the case of three equal masses). The proof relies on the construction of hyperbolic invariant sets whose dynamics is conjugated to that of the shift of infinite symbols (i.e., symbolic dynamics). That is, we construct invariant sets for the three body problem with chaotic dynamics, which moreover contain oscillatory motions.
This is a joint work with Pau Martin, Jaime Paradela and Tere M. Seara.

Zaher Hani (University of Michigan, Ann Arbor, MI, USA)

The mathematical theory of wave turbulence

Wave turbulence is the theory of non-equilibrium statistical mechanics for wave systems. Initially formulated in pioneering works of Peierls, Hasselman and Zakharov early in the past century, wave turbulence is widely used across several areas of physics to describe the statistical behavior of various interacting wave systems. We shall be interested in the mathematical foundation of this theory, which for the longest time had not been established. The central objects in this theory are: the “wave kinetic equation” (WKE), which stands as the wave analog of Boltzmann’s kinetic equation for interacting particle systems, and the “propagation of chaos” hypothesis, which is a fundamental postulate in the field that lacks mathematical justification. Mathematically, the aim is to provide a rigorous justification and derivation of those two central objects: this is Hilbert’s Sixth Problem for waves. In this talk, we shall describe some recent results with Yu Deng (University of Southern California) in which we give a full resolution of this problem, namely a rigorous derivation of the wave kinetic equation and a justification of the propagation of chaos, in the context of the nonlinear Schrödinger equation.

Sakshi Jain (Università di Roma Tor Vergata, Roma, IT)

Discontinuities cause essential spectrum

We study transfer operators associated to piecewise monotone interval transformations with a non-trivial discontinuity and show that the essential spectrum is large for any useable Banach space. Constructing a Banach space we show that this estimate is optimal in that there exists a space on which this best possible estimate on the essential spectral radius can be obtained.

Mark Kesseböhmer (Universität Bremen, Bremen, DE)

Spectral dimensions of Kreĭn–Feller operators in higher dimensions

We give a gentle introduction to Kreĭn–Feller operators for finite Borel measures ν on the d-dimensional unit cube via a form approach. We establish a connection between the spectral dimension of these operators and the notion of the spectral partition function and partition entropy of ν. Assuming that the lower ∞-dimension of ν exceeds $d-2$, we show that the upper Neumann spectral dimension coincides with the unique zero of the spectral partition function. Examples are given for the critical case, that is the lower ∞-dimension of ν equals $d-2$. We provide additional regularity assumptions on the spectral partition function, guaranteeing that the Neumann spectral dimension exists and coincides with the Dirichlet spectral dimension. The significance of our new approach is illustrated by several prominent examples—namely absolutely continuous measures, Ahlfors–David regular measure, and self-conformal measures with or without overlaps—for which the spectral partition function is essentially given by its L^q-spectrum and both the Dirichlet and Neumann spectral dimensions exist. This is joint work with Aljoscha Niemann.

Beatrice Langella (SISSA, Trieste, IT)

Growth of Sobolev norms in quasi-integrable systems: a quantum Nekhoroshev theorem

In this talk I will analyze an abstract linear time dependent Schrödinger equation of the form

$$i\partial_t\psi = (H_0 + V(t))\psi \tag{1}$$

with H_0 a pseudo-differential operator of order $d > 1$ and $V(t)$ a time-dependent family of pseudo-differential operators of order strictly less than d. I will introduce abstract assumptions on H_0, namely steepness and global quantum integrability, under which we can prove a $|t|^\varepsilon$ upper bound on the growth of Sobolev norms of all the solutions of (1). The result I will present applies to several models, as perturbations of the quantum anharmonic oscillator in dimension 2, and perturbations of the Laplacian on a manifold with integrable geodesic flow, and in particular: flat tori, Zoll manifolds, rotation invariant surfaces and Lie groups. The case of several particles on a Zoll manifold, a torus or a Lie group is also covered. The proof is based a on quantum version of the proof of the classical Nekhoroshev theorem.
This is joint work with Dario Bambusi.

Carlangelo Liverani (Università di Roma Tor Vergata, Roma, IT)

Asymptotic properties of deterministic fast-slow systems

Fast-slow systems are ubiquitous and are usually treated using averaging or homogenization theory. However, in many applications, one is interested in what happens for much longer times than the ones granted by averaging or homogenization. When the fast variable has a chaotic motion, fast-slow systems are a natural example of partially hyperbolic systems, so it seems possible to combine probability techniques with dynamical systems techniques to investigate longer times. A general theory is sorely missing. However, I will discuss a simple example showing that several results are within reach.

Jens Marklof (University of Bristol, Bristol, UK)

Geodesic random line processes and the roots of quadratic congruences

In 1963 Christopher Hooley showed that the roots of a quadratic congruence mod m, appropriately normalized and averaged, are uniformly distributed mod 1. In this lecture, which is based joint work with Matthew Welsh (Bristol), we will study pseudo-randomness properties of the roots on finer scales and prove for instance that the pair correlation density converges to an intriguing limit. A key step in our approach is to translate the problem to convergence of certain geodesic random line processes in the hyperbolic plane, which in turn exploits equidistribution properties of horocycle flows.

Jessica Elisa Massetti (Università di Roma Tre, Roma, IT)

On the persistence of periodic tori for symplectic twist maps

Invariant tori that are foliated by periodic points are at the core of the fragility of integrable systems since they are somehow extremely easy to break, in counterposition to the generic robustness of the quasi-periodic ones considered by KAM theory. On the other hand, the investigation of rigidity of integrable twist maps, i.e., to understand to which extent it is possible to deform a map in a non-trivial way preserving some (or all) of its features, is related to important questions and conjectures in dynamics. In this talk I shall discuss the persistence of Lagrangian periodic tori for symplectic twist maps of the $2d$-dimensional annulus and a rigidity property of completely integrable ones.
This is based on joint work with Marie-Claude Arnaud and Alfonso Sorrentino.

Péter Nándori (Yeshiva University, New York, NY, USA)

Flexibility of the central limit theorem in smooth dynamical systems

We say that a diffeomorphism that preserves a smooth probability measure satisfies the CLT if the ergodic sums of all sufficiently smooth functions converge to a Gaussian low. Many diffeomorphisms are known to satisfy the CLT under the usual scaling $\sqrt{n}$. Most of these examples also share many other chaotic properties, such as ergodicity, mixing, positive entropy, K property, Bernoulli property. In this talk, I will present new examples of diffeomorphisms that satisfy the CLT but in a more exotic way. For example, the scaling may be regularly varying with index 1, or several ergodic properties from the above list may fail. All our examples fall into the class of generalized T, T^{-1} transformations. I will also discuss some other unusual properties of this class of dynamical systems.
Based on joint work with D. Dolgopyat, C. Dong and A. Kanigowski.

Tariq Osman (Queen's University, Kingston, ON, CA)

The Jacobi theta function near the real line

We define generalised theta sums as exponential sums of the form

$$S_N^f(x;\alpha,\beta) := \sum_{n\in\mathbb{Z}} f(\tfrac{n}{N})\, e((\tfrac{1}{2}n^2+\beta n)x+\alpha n),$$

where $e(z) = e^{2\pi i z}$. If α and β are fixed real numbers, and x is chosen randomly from the unit interval, we may use homogeneous dynamics to show that $\frac{1}{\sqrt{N}}S_N^f$ possesses a limiting distribution as N goes to infinity, provided f is sufficiently regular. In joint work with F. Cellarosi, we prove that for specific rational pairs (α, β) this limiting distribution is compactly supported and that all other rational pairs lead to a limiting distribution with heavy tails. This complements existing work of F. Cellarosi and J. Marklof when $(\alpha, \beta) \in \mathbb{R}^2 \setminus \mathbb{Q}^2$.

Mark Pollicott (University of Warwick, Coventry, UK)

A hyperbolic dynamics perspective of flat surfaces

Geodesic flows on surfaces of negative curvature are excellent examples of dynamical systems where techniques from hyperbolic dynamics apply to show such things as estimates on growth (of closed orbits); equidistribution results; (entropy) rigidity results, etc. In this talk I want to consider how this viewpoint might be used to study certain properties of flat surfaces, which have zero curvature except at a finite number of (conical) singular points.

Manuel Quaschner (Friederich-Alexander-Universität, Erlangen-Nürnberg, DE)

Non-collision singularities and the (4+4)-system

The existence of non-collision singularities in the n-body problem was already conjectured by Painlevé in 1895. Even before the existence was proven in the 1990s, the question came up, whether the set of all initial conditions leading to non-collision singularities is a set of measure zero. A first result of this kind was proven for $n = 4$ particles in $d \geq 2$ dimensions by Saari (1977). Using the so-called Poincaré surface method, Fleischer (2018) could improve this for $n = 4$ particles in $d \geq 2$ dimensions by extending the result to a wider class of potentials. But the problem is still open for more than four particles. After an overview of these works, we will consider some special case of orbits with $n = 8$ particles that split into two subclasses of four particles each. If each of these subclasses diverges to spatial infinity and the subclasses are separated in a suitable way, we can prove an improbability result for the set of these orbits. Therefore some difficulties concerning the changes of some quantities of the subsystem (e.g., the energy and the angular momentum) have to be overcome.

Tanja Schindler (Universität Wien, Wien, AT)

A quantitative central limit theorem for certain unbounded observables over piecewise expanding interval map

Many limit theorems in ergodic theory are proven using the spectral gap method. So one of the main ingredients for this method is to have a space on which the transfer operator has a spectral gap. However, most of the classical spaces, like for example the space of Ḧlder or quasi-Hölder function or BV functions don't allow unbounded functions. We will give such a space which allows observables with a pole at the fixed points of a piecewise expanding interval transformation and state a quantitative central limit theorem using Edgeworth expansions. As an application we give a sampling result for the Riemann-zeta function over a Boolean type transformation.
This is joint work with Kasun Fernando.

Dmitry Turaev (Imperial College, London, UK)

On triple instability

If a dynamical system depends on three or more parameters, and if the dimension of the phase space allows for it, then the stability boundary (in the parameter space) for an equilibrium state or a periodic orbit can contain parameter values for which at least three characteristic exponents vanish simultaneously. In this case, the loss of stability leads to the birth of chaotic dynamics associated with the Shilnikov saddle-focus loop. We show that such bifurcations lead to chaos of the maximal possible complexity.

Sandro Vaienti (CPT, Luminy & Université de Toulon, FR)

A few results on quenched random dynamics on open systems

We consider random open dynamical systems. We obtain a first-order perturbation formula in this setting, which allows us to treat various quantities like the escape rate, equilibrium states, limit theorems. Our new machinery is then deployed to create a spectral approach for a quenched extreme value theory that considers random dynamics with general ergodic invertible driving, and random observations.
In collaboration with J. Atnip, G. Froyland and C. Gonzalez-Tokman.