Between discrete and continuous structures.

A long standing theme in applied analysis and PDEs is to understand how to connect finite particle models with finite degrees of freedom to continuum differential models with infinite degrees of freedom. In mathematics, the numerical analysis and approximation of PDEs fits into this scheme. In physics, examples are given by finite particle approximations of interacting systems of infinite particles. In most of these theories it is the continuum limit which pre-determines the construction of the finite approximations. In our lectures we take the approach of introducing finite dynamics from the outset and leave open the determination of suitable continuum limit dynamics. Our motivation to reverse the point of view comes from variational fractals models. In this case finite number of interactive particles come first in the construction, the limit as the number of particles becomes infinite being then regulated by suitable scaling laws. Continuum limits of physical significance can be attained which cannot be described in terms of differential operators.

Our presentation is aimed at illustrating some simple examples of such discrete-to-continuum transitions and related concepts and tools. The presentation is split in four parts:

1. *Variational convergence.* The transition of finite discrete dynamics into dynamics with infinite degrees of freedom in many cases can be formulated in variational terms as a convergence problem for suitable sequences of energy functionals. We describe this functional convergence setting and its connection with duality, operator convergence and convergence of spectra.

2. *Variational fractals.* We give various examples of energy forms on fractals obtained in the variational limit of energy forms on finite pre-fractal graphs, suitably scaled. We present non self-similar constructions and limits obtained by multiple scaling and describe the convergence of spectral measures and the non-differential limit operators. A rigorous quasi-metric setting for the power-law scaling formalism of physics based on abstract harmonic analysis is outlined.

3. *Impulsive curve growth.* We introduce generalized ODEs equations and Cauchy initial value problems that generate trajectories and orbits with fractal geometry. The motion of curves governed by these ODEs has an impulsive character in space-time and is governed by discrete force fields. We apply the theory to describe the growth of curves that fill open regions of space and to construct small open sets with large boundaries of fractal geometry. External parameters can be built in the models in view of applying optimal control techniques to the optimal design of boundaries and interfaces.

4. *Self-organized-criticality.* We introduce a purely discrete model for the self-organized-criticality theory of sand pile type. The purpose of this construction is to bridge the gap between the automata models of physics and the recent mathematical theory in terms of differential equations. The model is based on synchronization of space and time. Initial value problems for nonlinear difference equations of obstacle type on an increasing sequence of spatial grids are coupled with a time impulsive system of jumps for the initial states on the grids that takes into account time actualization. The model captures the essential feature of self criticality, namely, the equilibrium is attained in a finite time.