Sergio Benenti

How to compute the age of the Universe

Abstract:
If you know the current values of the Hubble parameter and of the cosmological constant, then a "magic" formula gives you an estimate of the age of the Universe which is perfectly consistent with that provided by cosmologists. This formula is one of the main results provided by an axiomatic approach to cosmology based on simple and reasonable assumptions, including the cosmological principle of homogeneity and isotropy.
Reference: http://arxiv.org/abs/1605.06260

Alexey Bolsinov

Jordan-Kronecker invariants of finite-dimensional Lie algebras

Abstract:
For any finite-dimensional Lie algebra we introduce the notion of Jordan-Kronecker invariants and discuss their properties and examples. These invariants naturally appear in the framework of the bi-Hamiltonian approach to integrable systems on Lie algebras and are closely related to the argument shift method. The idea is very naive. Take two compatible Poisson brackets $\{\ ,\ \}_0$ and $\{\ ,\ \}_1$ on a smooth manifold M and consider them at a fixed point $x \in M$ as just two skew-symmetric forms A_0 and A_1 on $V = T^*_x M$. Such a pair of skew symmetric forms (matrices) can be characterised by its algebraic type, i.e. a collection of discrete invariants describing the structure of their simultaneous canonical form known as the Jordan-Kronecker decomposition. This algebraic type remains the same on an open everywhere dense subset $U \subset M$ and can be understood as the algebraic type of the Poisson pencil $\{\ ,\ \}_0 + \lambda\{\ ,\ \}_1$ at generic points on M.
On the dual space $\mathfrak{g}^*$ of a Lie algebra $\mathfrak{g}$ there are two standard compatible Poisson brackets

$$\{f, g\}(x) = \sum c^k_{ij} x_k \frac{\partial f}{\partial x^i}\frac{\partial g}{\partial x^j} \quad \text{and} \quad \{f, g\}_a(x) = \sum c^k_{ij} a_k \frac{\partial f}{\partial x^i}\frac{\partial g}{\partial x^j},$$

where $a \in \mathfrak{g}^*$ is a fixed element. The Jordan–Kronecker invariant of $\mathfrak{g}$ is defined as an algebraic type of the Poisson pencil generated by these brackets for generic $x, a \in \mathfrak{g}^*$ (notice that this definition only uses the structure tensor c^k_{ij} of $\mathfrak{g}$ and no other additional structure). The aim of the talk is to explain how this naive idea and these invariants can be used for studying Lie algebras and their polynomial invariants.

Francesco Calogero

Solvable dynamical systems and generations of (monic) polynomials

Abstract:
Novel classes of dynamical systems — solvable by algebraic operations — will be reported, including a technique to identify endless hierarchies of such Newtonian (accelerations equal forces) problems describing an arbitrary number of nonlinearly interacting point-particles moving in the complex plane — as functions of continuous or discrete time. And the related notion of generations of (monic) polynomials will be introduced and discussed. Part of this work has been done with Oksana Bihun and with Mario Bruschi.

Roberto Camassa

Some fundamental issues at the boundary of math and fluids

Abstract:
Fluid dynamics (and continua in general) can be given an axiomatic structure that makes for a consistent mathematical theory. The extent to which some of the underlying mathematical assumptions

reflect realistic physical properties of fluid behavior, and the inherent limitations of their unavoidable idealizations, is perhaps best illustrated by the evolution of fluids interacting with boundaries. This and the companion talk by G. Ortenzi will review elements of the theory and discuss some of the most relevant features encountered in modeling fluids under these circumstances. (Based on collaborative work with Gregorio Falqui, Giovanni Ortenzi and Marco Pedroni).

Alberto De Sole

Classical affine W-algebras for gl_N and associated integrable hierarchies

Abstract:
I will describe a new method of integrability for Hamiltonian hierarchies of PDEs based on the notion of Adler type pseudodifferential operator and the theory of quasideterminants. I will then show how to use this method to construct an integrable hierarchy for the W-algebra of gl_N and an arbitrary its nilpotent element.
Joint work with V. Kac and D. Valeri.

Boris Dubrovin

Computing correlators from integrable systems

Abstract:
A simple and efficient procedure of computing logarithmic expansion of tau-functions of integrable systems will be explained. It will be applied to computing the Witten-Kontsevich intersection numbers on the moduli spaces of algebraic curves as well as the correlators of Hermitian random matrices. The talk is based on a series of joint papers with Marco Bertola and Di Yang (2015-16).

Gregorio Falqui

Two-layer interfacial flows beyond the Boussinesq approximation: a Hamiltonian approach

Abstract:
The theory of integrable systems of Hamiltonian PDEs is used to discuss evolution equations resulting from vertical-averages of the Euler system for two-layer stratified flows in an infinite 2D channel. Long-wave asymptotics together with the Boussinesq approximation of neglecting the fluids' inertia is then applied to reduce the leading order vertically averaged equations to the shallow-water Airy system, and thence to the dispersionless non-linear Schroedinger equation. The full non-Boussinesq system for the dispersionless limit can then be viewed and studied as a deformation of this well known equation.

Evgeny Ferapontov

Dispersionless integrable hierarchies and GL(2, R) geometry

Abstract:
$GL(2, R)$ geometry on an n-dimensional manifold M is defined by a field of rational normal curves of degree $n-1$ in the projectivised cotangent bundle of M. Such geometry is known to arise on the moduli spaces of solutions to ODEs with vanishing Wunschmann invariants. In this talk I will describe yet another natural source of $GL(2, R)$ structures, namely integrable hierarchies of the dispersionless Kadomtsev-Petviashvili type. In the latter context, $GL(2, R)$ structures coincide with the characteristic variety (principal symbol) of the hierarchy. Dispersionless hierarchies provide explicit examples of various particularly interesting classes of $GL(2, R)$ structures studied in the literature. Thus, we obtain torsion-free $GL(2, R)$ structures of Bryant which appeared in the context of exotic holonomy in dimension four, as well as totally geodesic $GL(2, R)$ structures of Krynski: the latter possess a compatible affine connection (with torsion) and a two-parameter family of totally geodesic

manifolds (coming from dispersionless Lax equations), which make them a natural generalisation of the Einstein-Weyl geometry. Our results imply, in particular, that ODEs with vanishing Wunschmann invariants are governed by a dispersionless integrable system, the fact which is apparently new. (Based on joint work with Boris Kruglikov).

Jean-Pierre Françoise

Integrable Hamiltonian Systems and Elliptic Fibrations

Abstract:
Some families of Hamiltonian Systems are integrable by Elliptic Functions. We focus on the case, where it is possible to associate to a family of IHS depending on parameters an Elliptic Fibration in the sense of Kodaira. In such case, it is interesting to compare the symplectic invariants of the IHS near equilibrium points with local or global invariants associated with the Elliptic Fibration. Birkhoff normal form is a power series expansion associated with the local behavior of the Hamiltonian systems near a critical point. This Birkhoff normal form can be computed in terms of elliptic functions using relative cohomology methods for the pendulum and for the free rigid body dynamics (Gallavotti-Garrido-Françoise, 2010, 2013). To the free rigid body motion is naturally associated the Naruki-Tarama Elliptic Fibration. The monodromy of the Naruki-Tarama fibration can be determined by using analytic extensions as parameters vary of the Birkhoff normal form (Tarama-Françoise, 2015).

Darryl Holm

Can stochasticity prevent blow-up in the b-equation?

Abstract:
The deterministic b-equation, for a real constant b, is given by

$$m_t + um_x + bmu_x = 0\,,$$

where $m(x,t) = u - \alpha^2 u_{xx}$, with lengthscale α.
This is a nonlinear, nonlocal evolutionary PDE defined on the real line for a 1D velocity profile $u(x,t)$ which tends to zero at spatial infinity $|x| \to \infty$.

The b-equation has a steepening lemma for $b > 1$. According to this steepening lemma, the solutions of the initial value problem for the deterministic b-equation for $b > 1$ can blow up in finite time, in the sense of wave breaking. That is, an initial condition for $u(x,0)$ whose profile is a graph on the real line possessing an inflection point of negative slope will develop a vertical negative slope in finite time.
After discussing some of the other properties of the deterministic b-equation solutions, we will investigate whether inclusion of Stratonovich noise in a stochastic b-equation can prevent this blow up.
In the case $b = 2$, the dynamics follows from Hamiltons principle. With this additional structure, several further observations can be made about the effects of introducing stochasticity in nonlinear evolutionary PDE which may be of relevance in fluid dynamics.

Alberto Ibort

On a class of integrable Hamiltonian systems related to the optimal control problem of two coupled spins

Abstract:
We will describe a class of completely integrable nonlinear coupled Hamiltonian systems that arise from the description of an optimal control problem for two spin systems.

Yuji Kodama

Confluence of generalized hypergeometric functions and integrable hydrodynamic type equations

Abstract:
We start with a brief introduction of the Gelfand hypergeometric (GHG) functions and their confluences by means of the action of the centralizers of regular elements. The confluence then implies that the GHG functions are now defined in a degenerate cell of the Grassmannian. We construct integrable hydrodynamic type systems defined on such cells.
This is a joint work with Boris Konopelchenko.

Yvette Kosmann-Schwarzbach

Variations on Hamiltonian structures

Abstract:
Introduced by some historical remarks, my talk will be a quick survey of some of the current generalizations of Hamiltonian structures: Poisson structures with background flux and quasi-Poisson structures on manifolds with a group action.

Giuseppe Marmo

Information Geometry and Hamilton-Jacobi equation for divergence functions

Abstract:
Distinguishability functions on the statistical manifold of probability distributions are shown to be solutions of the Hamilton-Jacobi equation associated with a properly chosen Lagrangian. By replacing probabilities with probability amplitudes we argue that this approach holds true also in the framework of geometrical quantum mechanics.

Alexander Mikhailov

Symmetric squares of hyperelliptic curves, associated vector fields and automorphic Lie algebras

Abstract:
With the universal space $U_g \subset \mathbb{C}^{2g+5}$ of symmetric square of hyperelliptic curves we associate a graded ring

$$\mathcal{R} = \mathbb{C}[X_1, X_2, Y_1, Y_2, x_1, \dots, x_{2g+1}]/J, \ \deg x_j = \deg X_k = 2, \ \deg Y_k = 2g{+}1, \ k = 1,2; \ j = 1, \dots, 2g{+}1,$$

where $J = \langle \alpha_2, \pi_1, \pi_2 \rangle$ is the ideal generated by polynomials

$$\alpha_2 = \sum_{k=1}^{2g+1} x_k, \quad \pi_j = Y_j^2 - \prod_{k=1}^{2g+1} (X_j - x_k), \ j = 1,2, \tag{1}$$

and also the invariant subring $\mathcal{R}^G \subset \mathcal{R}$, where $G = S_{2g+1} \times S_2$ with respect to the group S_{2g+1} of permutations of the roots $x_1, \dots, x_{2g+1}$ and involution $(X_1, Y_1) \longleftrightarrow (X_2, Y_2)$. We have constructed a graded polynomial Lie algebra $\mathfrak{A}(\mathcal{R}^G)$ of derivations of the coordinate ring $\mathcal{R}$, such that $L : \mathcal{R}^G \mapsto \mathcal{R}^G, \forall L \in \mathfrak{A}(\mathcal{R}^G)$. In algebra $\mathfrak{A}(\mathcal{R}^G)$ we have found $2g$ operators $L_0, L_2, \cdots, L_{4g-2}$ which preserve the ideal $J_\Delta \in \mathcal{R}^G$ generated by the discriminant Δ of the hyperelliptic curve. Moreover, taking the localisation $\mathfrak{A}(\mathcal{R}^G[(X_1 - X_2)^{-2}])$, we have found two additional operators $L_{2g-3}, \ L_{2g-1}$ which are commuting and annihilate functions of variables $x_1, \dots, x_{2g+1}$. It turns out that these $2g + 2$ operators generate $\mathcal{R}^G$–polynomial Lie algebra $\mathfrak{A}_g$. It is remarkable that two commuting vector fields $L_{2g-3}, \ L_{2g-1}$ can be found using a universal construction on symmetric squares of plane curves

Lemma 1 *Let $F(X, Y)$ be a twice differentiable function and derivations D_k be defined as*

$$D_k = \partial_{Y_k}(F(X_k, Y_k))\partial_{X_k} - \partial_{X_k}(F(X_k, Y_k))\partial_{Y_k}, \qquad k = 1,2.$$

Then the vector fields

$$\mathcal{L}^1 = \frac{D_1 - D_2}{X_1 - X_2}, \qquad \mathcal{L}^2 = \frac{X_2 D_1 - X_1 D_2}{X_1 - X_2}$$

commute*, have function F in the kernel $\mathcal{L}^i(F(X_j, Y_j)) = 0$ and map symmetric $(X_1, Y_1) \leftrightarrow (X_2, Y_2)$ functions into symmetric.*

Ring $\mathcal{R}^G$ we identify with the coordinate ring of U_g. We show that there is a canonical polynomial map $\varphi : \mathbb{C}^{2g+2} \mapsto U_g$ which induces the coordinate rings homomorphism $\varphi^* : \mathcal{R}^G \mapsto \mathcal{A}$, where $\mathcal{A}$ is the coordinate ring of $\mathbb{C}^{2g+2}$. The homomorphism φ^* is not isomorphism, but it became isomorphism after the localisation of $\mathcal{R}^G$ at $(X_1 - X_2)^{-2}$.
One of our main results is that left $\mathcal{A}$–module $\mathcal{A} \otimes_{\varphi^*} \mathfrak{A}_g$ can be represented as $\mathcal{A}$–polynomial Lie algebra of derivation of the polynomial ring $\mathcal{A}$. In the case $g = 2$ we have established explicitly the isomorphism of the Lie algebra $\mathcal{A} \otimes_{\varphi^*} \mathfrak{A}_2$ with the Lie algebra of polynomial vector fields on the universal space of Jacobians of genus 2, the later is given in terms of two–dimensional σ functions.
Representing the group G in the group Aut $(\mathcal{R} \otimes \mathfrak{G})$, where $\mathfrak{G}$ is a simple Lie algebra we obtain a new class of automorphic $\mathcal{R}^G$ Lie algebras with derivations $\mathfrak{A}_g$.
This is a joint work with Victor M. Buchstaber. We are grateful to the Royal Society International Exchanges Scheme grant for supporting this research.

Oleg Mokhov

On compatible and almost compatible metrics

Abstract:
We present some new results in the theory of compatible and almost compatible metrics (see [1–6]), which is motivated by Franco Magri's theory of compatible Poisson brackets and applications to integrable systems.
The work is supported by the Russian Science Foundation under grant 16-11-10260.
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Giovanni Ortenzi

Interface-boundary interplay in two layer Euler fluids

Abstract:
In this talk we shall discuss some results on the fluid interface motion. In particular, we will speak about models of an Euler fluid in a domain with boundaries in different bouyancy regimes. We shall show how the connection of the boundary with the interface can change substantially the fluid behavior. This is based on joint works with R. Camassa, G. Falqui, B. Konopelchenko and M. Pedroni.

Andrei Pogrebkov

Highest Hirota equations and their consequences

Abstract:
Relation of integrability with commutators identities on associative algebras is used to construct higher Hirota difference equations. Differential-difference and differential integrable equations that result from continuous limits of the higher Hirota difference equation are presented.

Orlando Ragnisco

Classical and quantum (super-)integrable systems on curved spaces: algebra helps a lot, but is not enough

Abstract:
The so called Taub-Nut and Darboux systems are probably the simplest examples of maximally superintegrable systems on curved spaces. Both belong to the Perlick's family, and specifically to the subclass of the so called "intrinsic oscillators". They are characterized by a single real parameter, and both the classical and the quantum versions are exactly solvable, and in a nice way, for a "natural choice" of the sign of such parameter. However, for the opposite sign, the situation changes dramatically: the kinetic energy is no more positive definite, and the problems look quite unphysical. To overcome this drawback we decided to change the sign of the hamiltonian (time-reversal), so that in the previously forbidden region of the phase space the systems are now confined, with all orbits closed, in the classical case, and exhibit a purely discrete spectrum in the quantum case. But the boundary condition that ensures square summability "destroys" exact solvability. . .

Tudor Ratiu

Stochastic reduction of classical variational principles

Abstract:
Reduction of Hamilton's variational principle for systems whose configuration space is a Lie group yields the Euler-Poincaré equations, first derived by Poincaré in 1901. These have been vastly extended to include the motion of advected quantities, both linear and affine, non-commutative versions thereof, as well as generalizations to problems whose configuration space is an arbitrary manifold and the Lagrangian is invariant under a Lie group action and the extension of all these theories to higher order Lagrangians. All the above mentioned systems, both in the smooth and discrete versions, should have various stochastic analogues, depending on what phenomenon is modeled. The basic idea is to start with variational principles, motivated by Feynman's path integral approach to quantum mechanics and also by stochastic optimal control. In this talk we concentrate only on systems with linear advected quantities, but in a stochastic setting. Replace in the material representation the deterministic particle paths by their perturbation with a Brownian motion and look at the mean paths in the directions determined by the drift vector field, subjected to a dispersion due to the Brownian motion effect. The question answered in this talk is the following: is there a deterministic dissipative equation, derived from a variational principle, that governs this motion? For the classical Euler-Poincaré equations, with the example of the periodic Euler equations for an ideal incompressible homogeneous fluid, as well as the generalization to geodesic flow on a Lie group, this has been carried out by Ana Bela Cruzeiro and her collaborators in the past few years. If advected quantities are present, different Brownian motions for each of them need to be introduced in order to take full advantage of all possible stochastic perturbations. The resulting equations are deterministic and dissipative. This explains the origin of dissipative terms in continuum mechanics models, such as viscous compressible fluid flow and dissipative magnetohydrodynamics. The general theory and these two examples are presented. Time permitting, a significant generalization that yields stochastic PDEs is also considered.

Paolo Maria Santini

The inverse spectral transform for integrable dispersionless PDEs: Cauchy problem, longtime behavior and wave breaking, and applications in physics

Abstract:
We present the main features of the inverse spectral transform (IST) of integrable dispersionless PDEs in multidimensions, stressing the main differences from the classical IST of soliton PDEs. We use such a novel IST to solve the Cauchy problem, construct the longtime behavior of solutions, and investigate analytically wave breaking phenomena, when they take place. This theory, applicable to several distinguished examples, like the dispersionless Kadomtsev - Petviashvili, the 2D dispersionless Toda, and the heavenly equations, has been developed in collaboration with S.V. Manakov, and this talk is dedicated to his memory. If time permits, we also discuss some rigorous aspects of this theory and the nonlocal nature of these equations, recently understood in collaboration with P.G. Grinevich and D. Wu, and/or applications to two Cauchy problems for nonlinear wave phenomena in Nature.

Vladimir Sokolov

Polynomial forms for quantum Calogero-Moser Hamiltonians and commutative subalgebras in universal enveloping algebras

Abstract:
We find transformations that bring the quantum Calogero-Moser Hamiltonians for $n = 2, 3, 4$ to differential operators with polynomial coefficients. These operators are related to special commutative subalgebras in the universal enveloping algebras of $gl(n + 1)$.

Ian Strachan

Applications of Novikov Algebras in (bi)-Hamiltonian Geometry

Abstract:
Linear bi-Hamiltonian structures — direct multicomponent generalizations of the bi-Hamiltonian structures of the celebrated KdV equation — were introduced by Gelfand and Dorfman and studied further by Balinski and Novikov. These are given in terms of an algebraic structure now known as a Novikov algebra. After a brief review of Novikov algebras and the resulting Hamiltonian structures, the basic structures are assembled in various ways to give Novikov algebra-valued KdV and Camassa-Holm equations. As a further application, a construction of homogeneous multi-component, multi-dimensional/first-order Dubrovin-Novikov Hamiltonian structures is given.

Giorgio Tondo

Haantjes Manifolds of KdV stationary flows

Abstract:
It is known that the Lax-Novikov-Dubrovin stationary flows of the KdV hierarchy are bi-Hamiltonian systems of Gelfand-Zakharevich type [1]. Moreover, a further reduction to the symplectic leaves of one of their two degenerate Poisson tensors provides systems of Stäckel type that admit symplectic-Nijenhuis structures, and generalized Lenard chains of gradients [2]. We will present some examples of such systems that admit, in their even-dimensional phase space, new geometrical structures [3], namely symplectic-Haantjes ones and Lenard-Haantjes chains [4, 5]. Such structures are reductions of Poisson-Haantjes structures living in the extended phase space of the Gelfand-Zakharevich systems.
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Johan Van de Leur

On the Extended Bigraded Toda Hierarchy

Abstract:
The (k, m)-Extended Bigraded Toda Hierarchy (EBTH) was introduced by Guido Carlet as a generalization of the Extended Toda Hierarchy (the case $k = m = 1$). The total descendant potential of $\mathbb{CP}^1$ with two orbifold points of order k and m is a tau function of this EBTH. In this lecture I will discuss the consequences of the String equation and the existence of additional symmetries and an Adler-Shiota-Van Moerbeke formula.

Alexander P. Veselov

Logarithmic Frobenius structures and hyperplane arrangements

Abstract:
The logarithmic Frobenius structures correspond to a class of solutions of the famous WDVV equation related to special finite sets of covectors called V-systems. The examples include Coxeter configurations and their restrictions, but the general classification of the V-systems is still an important open problem. I will discuss some relations of this problem with the theory of hyperplane arrangements, including holonomy Lie algebras and logarithmic vector fields. In particular, we will see that for a special class of V-systems called harmonic, which includes all Coxeter systems, the corresponding hyperplane arrangement must be free. In the irreducible Coxeter case the potentials of the corresponding gradient vector fields turn out to be Saito flat coordinates, or their one-parameter deformations. The talk is based on a joint work with M. Feigin.

Claude-Michel Viallet

On the integrability of discrete-time systems

Abstract:
To any discrete time system with a rational evolution one associates an index of complexity, defined from the asymptotic behaviour of the sequence of degrees of the iterates: the algebraic entropy. Vanishing of the entropy means integrability, non vanishing means chaos. The main feature is that the entropy does not take arbitrary values, with in particular the possible existence of an entropy gap, calling into question the notion of near integrability. I will present some results and conjectures.

Youjin Zhang

Bihamiltonian integrable hierarchies and their tau structures

Abstract:
Starting from a semisimple bihamiltonian structures of hydrodynamic type with certain additional properties, we show the existence of a Frobenius manifold structure and a tau structure for the associated bihamiltonian integrable hierarchy of hydrodynamic type (called the principal hierarchy). We then consider the classification of deformations of the principal hierarchy which possess tau structures.

Jorge P. Zubelli

Some Bihamiltonian Musings

Abstract:
Bihamiltonian hierarchies appear in a multitude of contexts ranging from water waves to the spectral theory of differential operators, and even in understanding perturbations of wave operators that preserve the strict Huygens principle. In this talk we shall explore some of the interesting connections between the bispectral property and the concept of bihamiltonian systems. The former was introduced by Duistermaat and Grunbaum and concerns the question of characterizing all differential operators that admit a family of eigenfunctions that are also eigenfunctions to a differential operator in the spectral variable. It is motivated by questions in limited angle computerized tomography and in orthogonal polynomials. Back in 1992, together with Franco Magri, we related the bispectral property of certain families of Schroedinger operators to nonlinear flows obtained by the second hamiltonian structure of the KdV hierarchy. More specifically, we described the manifolds of bispectral Schroedinger potentials by means of the flows of the hierarchy of master symmetries of the KdV. This turns out to lead to many interesting and seemingly unrelated problems, in particular to the (strict) Huygens property of certain wave operators. After a short motivation we shall review some of the more recent results in the area. Time allowing, we shall also discuss some completely integrable systems in the context of diffusion processes.